Principal Investigator Name: Michael W. Mislove
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Grant or Contract Title: Operational and Denotational Models for Languages Supporting Nondeterminism and Synchronization
Grant or Contract Number: N00014-91-J-1692
Reporting Period: 1 Oct 92 - 30 Sep 93

1) Productivity Measures:
Refereed papers submitted but not yet published: 4
Refereed papers published: 1
Unrefereed reports and articles: 0
Books or parts thereof submitted but not yet published: 1
Books or parts thereof published: 1
Patents filed but not yet granted: 0
Patents granted: 0
Invited presentations: 9
Contributed presentations: 0
Honors received: 4
Prizes or awards received: 0
Promotions obtained: none
Graduate students supported ≥ 25%: 0
Post-docs supported ≥ 25%: 0
Minorities supported: 0

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2) Detailed Summary of Technical Progress:

The goal of our research is to understand better the relationship between determinism and nondeterminism, and how this relationship can be reflected accurately in semantic models for high-level programming languages. The languages we focus on are *uniform languages*, such as CCS and CSP whose syntax is given in terms of uninterpreted atomic actions. Such languages are useful in understanding the way concurrent processes communicate and cooperate. Our approach is to explore the use of *spectral theory* to model the relationship between determinism and nondeterminism. The first case concerns *angelic nondeterminism*, which can be characterized by the property that deadlock is avoided whenever possible: if either branch of a nondeterministic sum of two processes can make progress, then progress is made. The power domain of interest is the *Hoare power domain*, $\mathcal{P}_{\mathcal{H}}(D)$, which is the family of non-empty Scott closed subsets of D in the usual containment order. It is well-known that the functor which associates to a domain D the family $\mathcal{P}_{\mathcal{H}}(D)$ is left adjoint to the forgetful functor from the category of algebraic lattices and sup-semilattice morphisms to the category of domains. Spectral theory provides an additional adjunction; the functor which associates to a completely distributive algebraic lattice its f nily of sup-primes gives rise to a right adjoint to the Hoare power domain functor (with its codomain suitably restricted). This forms an equivalence that shows that the "nondeterministically prime" elements completely determine the Hoare power domain, so, viewing the Hoare power domain as a model for nondeterminism, the functor which spectral theory provides gives a method to recover a model for the "nondeterministically prime" elements from the Hoare power domain (cf. eg, [MO91a], [MO91b]).

The second form of nondeterminism, *demonic nondeterminism*, is characterized by the property that deadlock is catastrophic. This semantics is used extensively in CSP, where the focus is on divergence, rather than deadlock. But the underlying principle remains the same, and it is characterized by the approach that, if either branch of the nondeterministic sum of two processes diverges, then the whole process diverges. Last year, we reported on the work of a Ph.D. student of mine, Han Zhang, who provided the mathematical underpinnings for an analysis of demonic nondeterminism similar to that which was provided in [MO91b] for angelic nondeterminism. Demonic nondeterminism is modeled using the *Smyth power domain*, which associates to an algebraic cpo D the family $\mathcal{P}_{\mathcal{S}}(D)$ of non-empty Scott-compact upper sets; this forms the free algebraic inf-semilattice over D. Zhang's results provide a method to recover a model of the sequential, deterministic processes in a language – namely, the set of inf-primes of the $\mathcal{P}_{\mathcal{S}}(D)$ – from the inf-semilattice $\mathcal{P}_{\mathcal{S}}(D)$, regarded as a model for a language which includes nondeterminism. Actually, Zhang's results hold for a much broader class of cpo's – the so-called quasi-continuous cpo's – and the problem which remained unsettled was to find a property which would single out which inf-semilattices have the property that their family of inf-primes forms a continuous cpo, instead of a quasi-continuous one. That result now is in hand, and it is quite simple to state. For a continuous inf-semilattice for which way-below is multiplicative and the inf-primes are prime separating (these form the general class of inf-semilattices which arise as the Smyth power domain of a quasi-continuous cpo), the inf-primes form a continuous cpo if and only if all primes are super primes, where p is *super prime* if $x \wedge y \leq p$ implies $x << p$ or $y << p$. Suitable restrictions produce the appropriate subcategories of algebraic objects. Thus, Zhang's theory shows that the category of domains is equivalent to the category of what he calls prime-separating arithmetic inf-semilattices in which all inf-primes are super prime. This assumption also is reasonable

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from a computational viewpoint, since it amounts to the hypothesis that compact elements in the model of the deterministic processes remain compact in the model for nondeterminism.

Having this result in place, we now plan to turn our attention to the third form on nondeterminism, *conventional nondeterminism.* This form of nondeterminism is characterized by the property that either branch of a nondeterministic sum can deadlock (or diverge) or make progress, independently of what the other branch does. This form of nondeterminism has traditionally been modeled using the *Plotkin power domain* under the *Egli-Milner* order. The goal is to find an analogue to the results for the Hoare and Smyth power domains which would allow one to recapture a model for the deterministic processes from within the Plotkin power domain, viewed as a model of conventional nondeterminism.

The second area where progress has been made this year is in our attempt to understand the relationship between the failures model and the failures-divergences model for CSP (cf. [BRH84, BR85]), on the one hand, and more traditional domain-theoretic approaches which the power domains exemplify, on the other. The goal is to discover in what sense the failures and failures-divergences models for CSP are universal. Last year we reported our results which show how the failures model can be captured as a quotient of a domain-theoretic construct. In the process, we constructed a new variant of the Plotkin power domain, which we call the *deterministic choice power domain.* In CSP, the "deterministic choice operator" $\Box$ is distinguished by the property that the environment can determine which branch of the choice is chosen on the first step of computation. This operator also is present in such languages as ACP, which is studied by the Dutch school. When properly analyzed, it becomes clear that the crucial issue in finding a model for this operator is that deadlock should be an identity for the operator $\Box$. Plotkin showed early on that there is no free semilattice with identity over a domain, but our construct shows there is such a semilattice over the category of pointed domains. It is this observation that leads us to the model of deterministic choice.

Our progress this year has been to incorporate the necessary ingredients into the domain-theoretic models we construct so that the failures-divergences model also can be expressed as a quotient of our construction. This work is still in progress. Our approach in both the case of the failures model and of the failures-divergences model is to begin with a model S for the sequential, deterministic fragment of CSP which we consider, and then to build a model supporting deterministic choice and, finally, nondeterministic choice over the sequential deterministic model. For example, the model for deterministic choice is gotten by taking the deterministic choice power domain $\mathcal{P}_{\Box}(\mathcal{S})$ of the model for the sequential deterministic sublanguage, and then the model for nondeterministic choice is gotten by taking the Smyth power domain $\mathcal{P}_{\mathcal{S}}(\mathcal{P}_{\Box}(\mathcal{S}))$ of the deterministic power domain just formed. The goal then isto show that the failures model, respectively, failures-divergences model, for CSP is a quotient of our construct. In fact, we have shown this is true for the failures model, where we take for S the family $A^* \cup A^*\delta \cup A^*\surd \cup A^\infty$ of finite words over the alphabet A of atomic actions, possible ending with δ (denoting deadlock) or $\surd$ (denoting normal termination), along with the infinite words over A. The motivation for the failures-divergences model is to distinguish the divergent process from the process which can perform any action but which still does not diverge. In this case, the proper model for the sequential, deterministic sublanguage is $S \times \{0, 1\}$, where those pairs whose second component is 0 represent the divergent processes. We also plan to explore the application of Abramsky's link [Abr88] between program logic and domain-theoretic models in this context, so that we can derive a canonical logic for the failures model and the failures-divergences model from our approach.

The research we have reported on so far is on untimed CSP, but we also have been engaged in work on Timed CSP. This work focuses on providing a general mathematical theory for deriving denotational models for Timed CSP and untimed CSP which support unbounded nondeterminism. The model for untimed CSP originally derived by Roscoe and Barrett, and the corresponding model for Timed CSP derived by Schneider led to the consideration of spaces which were neither complete partial orders (which were used to model untimed CSP) nor complete metric spaces (which were used to model Timed CSP).

Our work was based on the realization that a common thread which underlay both these constructions was a more general mathematical theory involving *local cpo's.* These are partially ordered spaces in which only those directed sets having an upper bound are guaranteed to have a least upper bound. One then uses a theory of dominating functions defined on related spaces where they are guaranteed to have fixed points to guarantee that all the operators from the language have least fixed points. This approach provides a completely abstract validation that recursive operators from the language have meanings in the denotational model which are given by least fixed points.

Our work also continues on a model for a less abstract dialect of Timed CSP. This more concrete language does not allow "instant prefixing," so that the process $a \rightarrow b \rightarrow STOP$ must take some small amount of time between the action a and the action b. This means that one must require any process which exhibits simultaneous actions to be able to execute them in either order; this language is somewhat harder to model than the more abstract one where instant prefixing is allowed. This work will be the focus of an anticipated visit by Schneider to Tulane this academic year.

Finally, we mention work on two disparate topics. The first is the problem of *lambda models*, which was first suggested to us by Samson Abramsky. The question is to determine which Cartesian closed categories contain a model of the untyped lambda calculus, and the particular category of interest is the category of Hausdorff k-spaces, which is the natural Cartesian closed category within the general category of Hausdorff spaces and continuous maps. Our results show that there are no non-degenerate compact Hausdorff models of the untyped lambda calculus [HM93]. In the process of writing up this result for publication, we have revisited the classic paper of Meyer [Mey82], where the results all were stated purely within the category of sets, and showed how to translate his results into various topological categories.

The second topic also is related to category theory. In the paper [MO93], we address the question of adjunctions between various categories of domains. This work resulted from the desire to understand whether there is a left adjoint to the inclusion functor from Scott domains into SFP-objects. The paper begins with a surprisingly simple counterexample, and then goes on to explore the various power domain constructs as left adjoints. Some of this is almost classical, in that it recaptures the traditional results about the Hoare, Smyth and Plotkin power domains. In addition, we explore a new power operator which is not a power domain, since the resulting object is not a semilattice, but instead is a free semigroup. The construction is inspired by a close examination of the Scott-closed sets, and is given on the object level as the family of bounded Scott-closed subsets of a domain.

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[MO93] Mislove, M. W. and F. J. Oles, Adjunctions between categories of domains, Special issue of *Fundamenta Informaticae* on Category Theory and Computer Science, accepted.

[MRS91] Mislove, M. W., A. W. Roscoe and S. A. Schneider, Fixed points without completeness, submitted.

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3) Publications, Presentations, Reports and Awards and Honors

a) Refereed papers submitted but not yet published:

1. Mislove, M. W., A topological algebra for angelic nondeterminism, *Theoretical Computer Science*, under revision (with F. J. Oles).
2. Mislove, M. W., Fixed points without completeness, submitted to the Proceedings, Eighth Workshop on MFPS, 46pp. (with A. W. Roscoe and S. A. Schneider).
3. Mislove, M. W., Adjunctions Between Categories of Domains, Special Issue of *Fundamenta Informaticae* devoted to Category Theory and Computer Science, accepted (with F. J. Oles).
4. Mislove, M. W., All Compact Lambda Models are Degenerate, Special Issue of *Fundamenta Informaticae* devoted to Category Theory and Computer Science, submitted (with Karl H. Hofmann).

b) Refereed papers published:

5. Mislove, M. W., Full abstraction and recursion, Proceedings of the REX Workshop on Concurrency, Lecture Notes in Computer Science **666** (1993), 384 – 397 (with F. J. Oles).

c) Unrefereed reports and articles:

None

d) Books or parts thereof submitted but not yet published:

6. Mislove, M. W., Proceedings of the Ninth Conference on the Mathematical Foundations of Programming Semantics, Lecture Notes in Computer Science, to appear, (with S. Brookes, M. Main, and A. Melton).

e) Books or parts thereof published:

7. Mislove, M. W., Proceedings of the Eighth Conference on the Mathematical Foundations of Programming Semantics, *Theoretical Computer Science* **111** (1993), 291 pp. (with R. Tennent).

f) Patents filed but not yet granted: none

g) Patents granted: none

h) Invited presentations:

1. Mislove, M. W., Domain-theoretic Models of CSP, Allgemeine Algebra Seminar, Technische Hochschule Darmstadt, October, 1992.
2. Mislove, M. W., Domain Theory, Spectral Theory and the Semantics of Nondeterminism, Festtagung für Karl H. Hofmann, Technische Hochschule Darmstadt, October, 1992.
3. Mislove, M. W., Recent and Not So Recent Results in Domain Theory, Programming Research Group, Oxford University, February, 1993.
4. Mislove, M. W., The Search for Lambda Models, 27th Annual Spring Topology Conference, Columbia, South Carolina, March, 1993.
5. Mislove, M. W., A Survey on Domain Theory and Its Applications, Vanderbilt University, March, 1993.
6. Mislove, M. W., Domain-theoretic Models of CSP, MASK Workshop, CWI, Amsterdam, April, 1993.
7. Mislove, M. W., Measure Algebras of Locally Compact Semigroups, Mini-Conference on Harmonic Analysis, University of Wollongong, Australia, June, 1993.
8. Mislove, M. W., A Survey of Applications of Domain Theory, Annual Meeting of the Australian Mathematical Society, University of Wollongong, July, 1993.

9. Mislove, M. W., The Search for Lambda Models, Seminar on Category Theory and Computer Science, Schloß Dagstuhl, Germany, July, 1993.

j) Honors received:

1. Mislove, M. W., Co-chairman, Ninth Conference on the Mathematical Foundations of Programming Semantics, Tulane University, New Orleans, April 7 - 10, 1993.
2. Mislove, M. W., Program Committee, Ninth Conference on the Mathematical Foundations of Programming Semantics, Tulane University, New Orleans, April 7 - 10, 1993.
3. Mislove, M. W., Co-chairman, Tenth Workshop on the Mathematical Foundations of Programming Semantics, Kansas State University, Manhattan, March 20 – 23, 1994.
4. Mislove, M. W., Co-chairman, Conference on Semigroups and Their Applications Honoring A. H. Clifford, Tulane University, March 27 – 30, 1994.

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4) Transitions and DoD Interactions:

We have reported in the research summary on the joint work which has evolved between the Principal Investigator and the group at the PRG, Oxford. This work focuses on using the techniques we have developed to craft new models for the various dialects of Timed and untimed CSP, and to understand better the existing models.

5) Software and hardware prototypes: none

6) Photographs, vugaphs or videotapes of work: none